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Normal and Weibull Distribution of Teak (*Tectona grandis*) Linn F. in Oke-Eri Plantation, Ogun State, Nigeria

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Abstract
Tree diameter distributions has contributed positively in stand management. They are frequently used for describing forest stand structure. The aim of this study was to evaluate the effectiveness as well as compare the accuracy of both the Normal and Weibull distributions in describing diameter distribution of Tectona grandis stands in Oke-eri plantation in Ogun state. Fifteen (15) temporary sample plots of 20 x 20m were laid in the plantations with the age series of 11, 12 and 13 years using stratified random sampling technique. Complete enumeration and measurement of all trees were carried out in each sample plot. The variable measured was diameter at breast height (DBH). The two-parameter (2-P) Weibull distribution was estimated using the method of Maximum likelihood estimation (MLE). The class range of diameter distribution was developed and the DBH were fitted into the Normal and Weibull distributions before being compared. The Normal distribution slightly fitted the data but the Weibull distribution gave the best fit. The two distribution functions were compared on the basis of their bias, Mean Absolute Error (MAE) and Root Mean Square Error (MSE) [0.048, 0.096.and 4.123; 0.016, 0.064 and1.011] respectively for Normal and Weibull distributions. The results from their comparison indicated that the Weibull distribution performed better than the Normal distribution. Hence, the 2-P Weibull distribution function estimated using the MLE method were adequate for modelling distribution diameter at breast height distribution in Oke-Eri plantation for sustainable management.
Keywords: *Diameter distribution, modelling, Maximum likelihood estimation, Normal and Weibull distributions*

Introduction

Diameter distributions are well known and has been extensively used for describing forest stand diameter structure (Duan *et al*, 2013). Fonseca *et al.,* (2009) stated that detailed information on the variability of tree diameters in natural and plantation forests are essential keys to sustainable forest management and are important to assess carbon sequestration in forest ecosystems. Tree diameter distributions also has key role in stand management. Diameter distributions also provide relevant information about stand structure, age structure, stand stability, etc. which are very vital in planning of silvicultural treatments. Tree diameter serves as an important factor in harvesting because it determines the type of machines used and the performance during felling as well as transportation of the wood.

Sghaier and Palm (2002) noted that at stand level, diameter distribution data can be a more scientific basis of tree selection for harvest. The parameter prediction method or recovery method can be used to obtain diameter distribution parameters at stand level (Palahi *et al*., 2006). Representative trees known as sample trees may then be drawn from the predicted distribution which can be further used to simulate forest stand development based on the tree-level growth models (Palahi *et al*., 2006). Literature searched showed that the Weibull distribution function was used for the first time in modeling diameter distributions of plantation stands (Bailey and Dell, 1973), and since then it has been used enormously in many growth models based on diameter distributions due to its flexibility and simplicity (Maltamo *et al*., 2000; Zhang *et al*., 2003; Liu *et al*., 2004).

The knowledge of diameter distributions is highly useful to describing and analyzing the structure of forest stands. It also serves the purpose of estimating age distribution, determining stand stability and computation of the number of trees in each diameter class for effective planning of silvicultural treatments (Carretero and Alvarez, 2013).

Diameter distributions will enable us know the density of small trees in a stand which can be regenerated to replace the current population of bigger trees and aid in evaluating the forest potential for sustainable management (Aigbe and Omokhua, 2014). Diameter distribution of trees in a specific stand is pivotal for studies of growth and yield of stands (Burnham and Anderson, 2002). In forest management, forest growth and yield models play a key role in forest growth processes and predicting forest growth. The diameter distribution is a key method to describe the uniformity and growth of a stand studies. Despite the evident efficiency of diameter distribution, there is dearth of information on modelling the diameter distribution of the Teak (*Tectona* grandis) plantation in Oke-Eri for forest management planning. Normal distribution is a random variable X, assuming all real values from $\alpha<X<\alpha$ has a normal distribution with its probability density function of the form (Clutter *et al*., 1983):

$$f(x) = \frac{1}{2}\pi r exp \frac{\frac{1}{2}(x-\mu)^2}{\sigma} \qquad (-\alpha<X<\alpha)\text{--------------------------equation1}$$

Where: π = pie (3.142), σ = standard deviation, μ = general mean and exp = exponential constant..

Teak's high oil content, high tensile strength and tight grain make it suitable in locations where weather resistance is necessary. It is used in the production of boat decks and also used for cutting boards, counter tops and as a veneer for indoor furnishings, it has also been said to be durable even when not treated; this is because of its natural oil that makes it resistant to termite and pest attack. It can cause acute blunting on edged tools due to silica content in the wood though it can be easily worked on. With time teak can weather to a silvery- grey finish, due to exposure to sunlight.

Materials and Methods

The research was carried out in Oke-Eri Teak Plantation; a private plantation owned by BISROD Furniture Company limited, located at the Northern part (Ijebu North) of Ijebu- ode Local Government Area, Ogun State, Nigeria. Ijebu- Ode is a Local Government Area located in the South- western part of Nigeria. Its lies between latitude 6^0 58′ 0” N and longitude is3^0 55′ 59" E of the equator. Restrictive random sampling was used for data collection in the plantation. The different ages of the teak plantation constituted the various strata. Sample plots of 20 x 20m were laid in each stratum and a complete enumeration of all trees taken. The variable measured was diameter at breast height which was measured at 1.3m above the ground. This was measured with the aid of diameter girth tape. The plantation was partitioned into 3 strata, based the on age criterion (2006, 2007, and 2008). Data was then collected within the sample plot size of 20m x 20m (0.04ha). The two diameter distribution functions tested were normal and Weibull distribution functions.

The equation for Normal distribution function used in this research is;

$$f(x) = \frac{1}{2}\pi r exp \frac{\frac{1}{2}(x-\mu)^2}{\sigma} \quad (-\alpha<X<\alpha)\text{ ------------------equation 2}$$

Where,

$\pi = pie\ (3.142)$

$\sigma = Standard\ deviation$

$\mu = General\ mean$

The probability density function of the two- parameter Weibull distribution function used is defined as:

$f(x)$ **= (c/b) (x/b) exp -(x/b)^{c}** **(x ≥ 0, b >, c > 0)** ---------------equation 3

The cumulative density function of this two- parameter function is given as:

$f(x)$ **= 1- exp** $\left[\mathbf{-(x/b)^{c}} \right]$ ---------------equation 4

Where,

$b = Scale\ parameter$

$c = Scale\ parameter$

x= Random variable (diameter)

exp = Exponential constant

The parameters for the normal distribution function are the arithmetic mean and standard deviation was estimated by using the following equations;

$$\widehat{m} = \frac{1}{n}\sum_{i=1}^{n} x_i$$ **--------------------** equation 5

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}[\![(x_{i-}\ \ \widehat{m}]\!])^2}$$ **-------------------**equation 6

For the Weibull distribution, the probability function of the two- parameter model was used, where the location parameter represented as (a)was equal to zero, (i.e. a = 0). The other parameters (b and c) were the scale and shape parameters respectively. The parameters were estimated using the maximum likelihood estimation method (MLE).

The parameters were estimated using the maximum likelihood method, after which they were fitted to the distribution functions. The class probabilities (*P*i) was obtained from it and afterword used to compute the diameter-class frequencies for each plot.
Predicted Number of tree per class (Ni) = N *x* Pi……………………………equation 7
Where: Ni = estimated tree number per class, N = tree number per ha and Pi = class probability.

Results and Discussion

Results

The results of the analysis of diameter at breast height and diameter distributions are shown below. This was based on the comparison of their probability density functions. The descriptive statistics is shown and represented in table 1.

Table 1. Descriptive statistics of the Diameter at breast height

Descriptive Measurement	Value
Mean	50.5cm
Standard Error	0.69
Median	49
Mode	55
Standard Deviation	14.51
Sample Variance	210.55
Kurtosis	3.13
Skewness	1.45
Coefficient of variation	0.29
Confidence Level (95.0%)	1.35

Table 1 shows the descriptive statistics (mean, standard error, median, mode, standard deviation, variance of the sample, kurtosis, skewness, range, minimum value, maximum value, total sum of, count, largest, smallest and the confidence level) of dbh in the study area.

Mean gives the average number of trees from the total sample of trees. This was obtained by the addition of the total number of trees divided by the number of trees sampled. The value of the mean is 50.05. The value of the skewness was 1.45 while that of excess kurtosis is 3.13. High positive skewness and peakness means that considerable number of trees are concentrated in the lower diameter classes (Gadow, 1983).

The standard error (SE) of a statistic (usually an estimate of a parameter) is the standard deviation of its sampling distribution or an estimate of that standard deviation. It is considered statistical accuracy of an estimate, which equates equal to the standard deviation of the theoretical distribution of a large population of such estimates. The standard error for the sampling distribution from the data collected is 0.69 which shows the level of accuracy of the data and that the sample statistic is closer to the population

Median is obtained by listing out the total number of trees and then picking out the middle value, which is 49. Mode is the set of data that occurs more frequently which is 55 from the collected data, this implies that trees with the DBH of 55 cm are more among the sampled trees, it therefore indicates that trees with DBH of 55 can be harvested depending on the manager's objective for pole or timber production. Standard deviation (SD, also represented by the Greek letter sigma σ or the Latin letter s) is an indices used to evaluate the amount of variation or dispersion of a set of data values. When the data points tend to be close to the mean of the set, it indicates a low SD, while a high SD indicates that the data points are spread out over a wider range of values. The square root of variance is computed by considering the variation between each data point relative to the mean. The SD value is 14.51, indicating a low standard deviation, so the sampled data are close to the expected value which implies that the data collected is accurate and can be referred to for future reference. The coefficient of variation (CV) can be obtained by dividing the mean with the standard deviation. CV greater than one (1) indicates a relatively high variation, a CV less than one indicates a relatively low variation, from the data collected, the CV is 0.29 which is less than 1, so therefore the data collected is accurate.

Table 2. Parameters of the Normal and Weibull Distribution

Distribution	Method	Parameters	Values
Weibull	MLE	b	12.97
		c	5.42
Normal		$\hat{m}$	50.04
		σ	14.51

Table 2 shows the estimated parameters of the distribution functions. The normal parameter shows the estimated arithmetic mean and the standard deviation. The Weibull distribution show the estimate of the shape and scale parameters. These parameters where used to model the Normal and Weibull distribution functions. The maximum likelihood estimation (MLE) was preferred because of its precision and asymptotic minimum variance.

Table 3. Diameter at breast height class distribution of various ages

	Years			
DBH (cm)	**11yrs**	**12yrs**	**13yrs**	**Total**
0-19.9	6	7	1	14
20-39.9	76	47	35	158
40-59.9	120	53	37	210
60-79.9	37	15	10	62
80 and above	2	1	1	4
Total	241	123	84	448

Table 3 shows the diameter distribution classes and their individual frequencies per age group. From the table, the highest frequency which falls under the age 11 is 120 and is between the diameter class of 40 to 59.9 while the frequencies 37 and 76 are in the diameter range 60 to 79.9 and 20 to 39.9 respectively. A total of 241 was found under the age 11. The highest frequency of trees found under age 12 is 53 and fell between the diameter class ranges of 40-59.9, diameter classes 20- 39.9 and 60- 79.9 had frequencies 47 and 15 respectively. The total number of trees found under age 12 was 123. For age 13, the diameter class range of 40- 59.9 has the highest frequency of trees of 37. Diameter class range of between 20- 39.9 and 60- 79.9 had frequencies of 35 and 10 respectively and has a total 84 trees. From the table above, it shows that age 11 has the highest frequency of trees (210) while ages 12 and 13 have the lowest frequency of trees. The implication of this is that the forest is still undergoing regeneration and recruitment, which is vital indicators of forest health and vigour (Jimoh *et al*., 2011). The table also shows that the highest frequency of trees in all ages fall under the diameter class range of 40-59.9. So therefore felling trees under the diameter class range 40- 59.9 can lead to sustainable management of the plantation. Figure 1 shows the curves associated with these distributions.

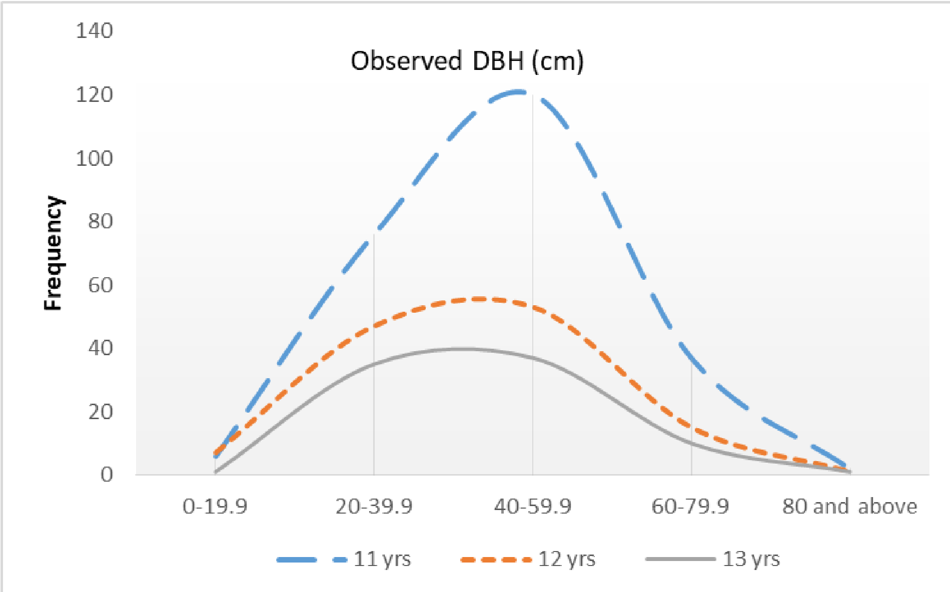

Figure 1. Curves of diameter classes distribution at various ages

Table 4 Distribution of Diameter at breast height using the Weibull distribution function

Weibull Dist. of Diameter Classes (cm)	**YEARS 11 yrs**	**12 yrs**	**13 yrs**	**Total**
0.800-0.820	1	2	0	3
0.821-0.840	17	14	11	42
0.841-0.860	128	68	46	242
0.861-0.880	94	39	27	160
0.881 and above	1	0	0	1
Total	241	123	84	448

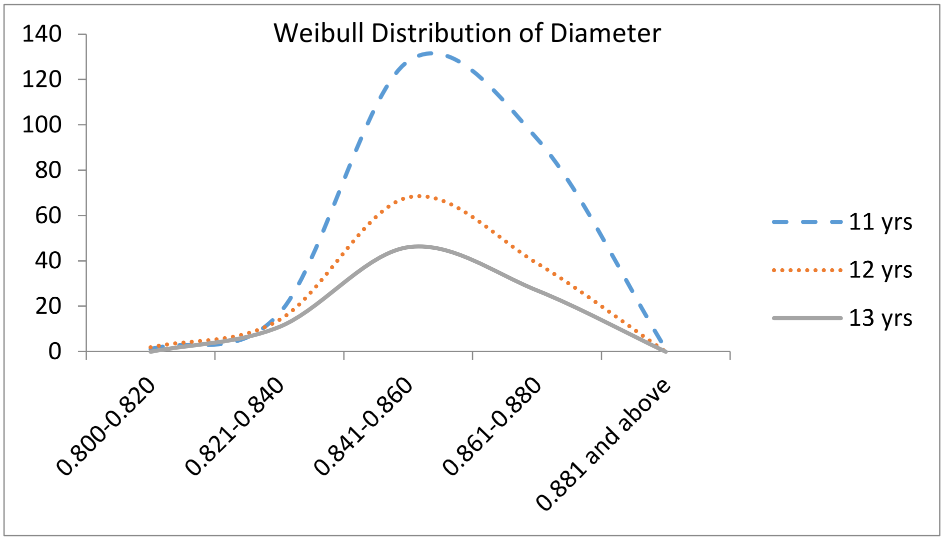

Figure 2. Curve of Weibull distribution of diameter at breast height

Table 4 and figure 2 shows the Weibull distribution of DBH classes and this is based on the probability function. The probability of having a tree falling between 0- 19.9 is 0.800-0.820 cm, Class 20- 39.9cm is between the ranges 0.821-0.840cm, and so the trend continues, i.e. diameter class range of trees of 80 and above falls under the Weibull distribution of 0.881 and above. The table indicates that at ranges 0.841-0.860cm, 127 is obtained as the highest frequency at age 11, this is in line with the diameter class range where age 11 has the highest frequency falling under the class range 40-59.9, This indicate the health and vigour of the forest. The data fitted perfectly into the Weibull distribution.

Table 5. Diameter at breast height (cm) distribution using the Normal distribution function

Normal Dist. Diameter Class (cm)	11 yrs	12 yrs	13 yrs	Total
0.005-0.009	1	2	0	3
0.01-0.049	7	7	2	16
0.05-0.099	10	7	9	26
0.1-0.499	98	58	40	196
0.5 and above	125	49	33	207
Total	241	123	84	448

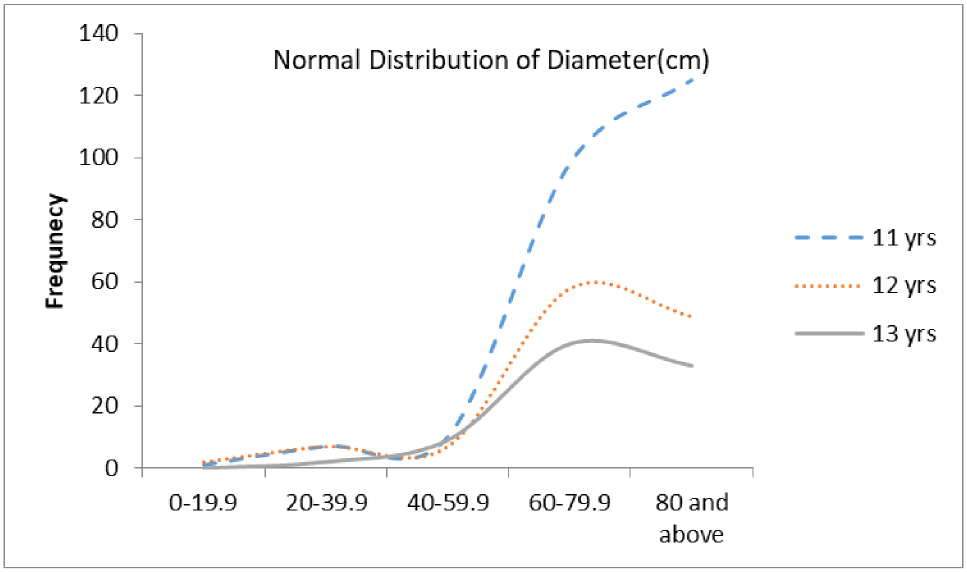

Figure 3. Curve of Normal Distribution of Diameter at Breast Height

For the Normal distribution, the distribution of diameter into classes are also based on the probability density function. The probability of having a tree under the diameter class ranges 0-19.9 is 0.005-0.009cm which was very low. Unlike the Weibull distribution that fitted data well, Normal distribution does fit but not effectively.

Table 6. Criteria used in the comparison of Weibull and Normal functions for diameter distribution

Comparison Criteria	Weibull	Normal
Bias	0.016	0.048
RMSE	1.011	4.123
MAE	0.064	0.0969

Table 6 shows the criteria used in the comparison of the Weibull and Normal distributions; the bias, Residual Mean Square Error (RMSE) and the Mean Absolute Error (MAE) of the diameter at breast height of trees obtained were used, they are used to measure how well your model performed or indicator for assessing the output quality. The table indicates that the Weibull function suits better for all the comparison criteria evaluated.

Table 7 shows the comparison of DBH data fitted in the Weibull and Normal distribution. According to the results of the Weibull distribution, diameter class 40 to 59.9cm had the highest frequency value of 242 trees which implies that majority of the trees as predicted by Weibull distribution have DBH ranging from 40 to 59.9cm. This means that trees under this class range can be sustainably utilized. On the other hand, for Normal Distribution, it was discovered that majority of the trees have DBH values of 80cm and above; therefore, for the Normal distribution, trees in the class range 80 and above can be sustainably utilized. Figure 4 further explains these results.

Table 7. Comparison of Weibull and Normal distribution functions of diameter classes

Diameter Classes (cm)	Frequency DBH	Weibull	Normal
0 - 19.9	4	3	3
20 - 39.9	100	42	16
40 - 59.9	233	242	26
60 – 79.9	101	160	196
80 and above	10	1	207
Total	448	448	448

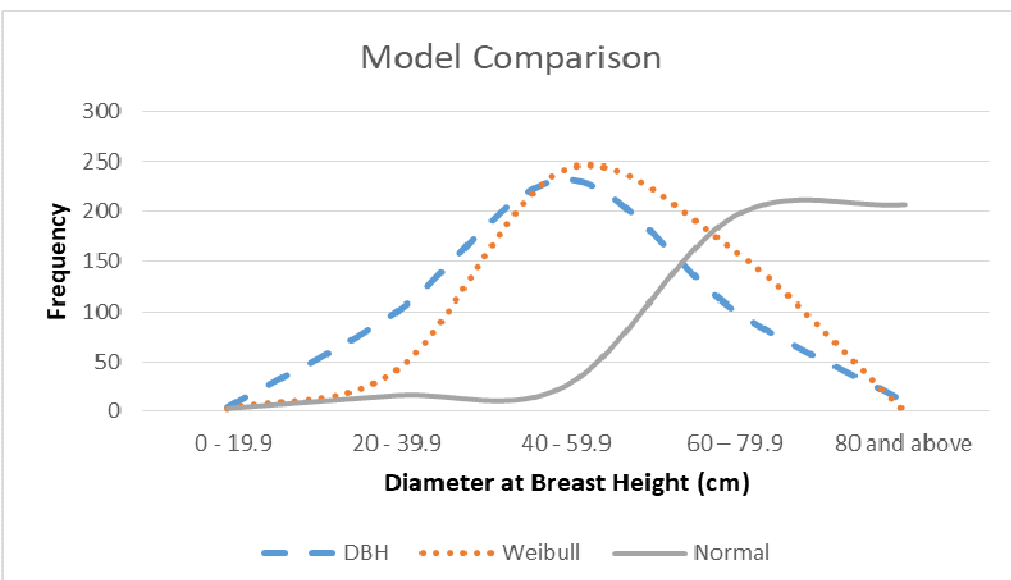

Figure 4. Graph showing the comparison of the distribution functions

Figure 4 shows that the Weibull Distribution gave the best fit from the two Distribution functions utilized in this study. This conforms to the findings of Sghaier *et al.* (2016) who worked on *Tetraclinis articulata* in Tunisia using the two parameter function.

The works of several researchers showed that the two-parameter Weibull function is a simple and accurate approach for modelling diameter distributions for *Pinus sylvestris* and *Picea abies* (L.) Karst. stands in Finland (Maltamo *et al*., 1995) for birch stands (*Betula alba L*.) in Spain, (Gorgoso 2003, Gorgoso *et al*. 2007). In addition, several similar studies had earlier used two-parameter Weibull function in a large number of related studies due to its flexibility and high correlation of its parameters with stand characteristics (Nogueira *et al*., 2005, Binoti *et al*., 2012). In this study, probability distribution was applied to estimate the diameter distribution, Weibull distribution was more flexible in fitting the diameter data in Oke-Eri Forest Reserve when tested with Kolmogorov smirnov Therefore, due to its flexibility and the very strong correlation between the probability density function (PDF) parameters and stand characteristics, the two-parameter Weibull distribution, using the maximum likelihood method, appears to be the most suitable for describing diameter distributions in this study. Diameter distribution model reveal structure of stand or forest and its application in scheduled treatment of silviculture stands in Erin-Oke forest reserve is recommended.

Conclusion

One of the several variables used to document the size of individual trees is diameter at breast height. This study has provided information on the diameter distribution of Oke-Eri Teak Plantation. The comparison on the effectiveness of the normal and 2-P Weibull distributions for characterizing the tree diameter of the plantation has been made. The Weibull function is shown to be the most suitable for fitting the diameter distribution of the plantation, this is because of its flexibility. The study reveals that a high proportion of trees falls under the diameter class range 40-59.9cm which indicates that they can be harvested for pole production purposes. Weibull distribution function is therefore recommended to diameter at breast height distribution of teak on Oke-Eri plantation because of its ability to present skewness and it also gave a better fit than the normal distribution function.

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